



A TECHNICAL PAPER FROM BODINE ELECTRIC COMPANY

## Introduction to Motor Constants for Fractional Horsepower Gearmotors

Motor constants are needed to calculate permanent magnet DC (PMDC) or brushless DC (BLDC) motor specifications and ratings, or to match the motor properly to an amplifier. The motor constants are required in order to predict the PMDC or BLDC motor's performance with changing variables, such as different input voltages or different loads (see Tables 1 and 2 for Bodine stock PMDC and BLDC motor constants). This application note explains what the constants are, how they are derived and how to use them.

### Common Motor Constants

The most commonly used motor constants are Torque Constant ( $K_t$ ), Voltage Constant ( $K_e$ ), Electrical Time Constant ( $\tau_e$ ), Mechanical Time Constant ( $\tau_m$ ), and Thermal Resistance ( $R_{th}$ ). Typical values for these constants are derived by using measured values of No Load Speed, No Load Current, Stall Torque, Circuit Resistance, Circuit Inductance, and Armature Inertia with the following equations:

**Torque Constant ( $K_t$ )** — describes the proportional relationship between torque and current.  $K_t$  is usually expressed in the units oz-in/Amp. See page 2 for additional information about torque constants.

$$K_t \text{ (oz-in./Amp)} = \frac{\text{Rated Torque (oz-in.)}}{\text{Rated Current (Amps)} - \text{No Load Current (Amps)}}$$

**Voltage Constant, or Back EMF Constant ( $K_e$ )** — is the Torque Constant expressed in different units, usually Volts/Krpm, in order to describe the proportional relationship between motor speed and generated output voltage when the motor is back driven as a generator in units of Volts/1000 rpm. See page 2 for additional information about voltage constants.

$$K_e \text{ (Volts/1000 rpm)} = K_t \text{ (oz-in./Amp)} \times .74$$

**Electrical Time Constant ( $\tau_e$ )** — is the time required for a motor to reach 63.2% of its stall current after applying a test voltage with the motor shaft locked. It is usually expressed in milliseconds. Applied Voltage equals Rated Current multiplied by Circuit Resistance

$$\tau_e \text{ (msec)} = \frac{\text{Circuit Inductance (mH)}}{\text{Circuit Resistance (Ohms)}}$$

[ $\tau_e$  = Time for current to reach 63% of final value]



Bodine type 24A PMDC and type 34B BLDC motors are used to power a range of Bodine gearmotors

**Mechanical Time Constant ( $\tau_m$ )** — is the time required for an unloaded motor to reach 63.2% of its no load speed after applying its rated voltage. It is usually expressed in milliseconds.

**For PMDC Motors:**

$$\tau_m \text{ (msec)} = \frac{R \text{ (Ohms)} \times J \text{ (oz-in-sec}^2\text{)}}{K_t \text{ (oz-in/Amps)} \times K_e \text{ (V/Krpm)}} \times 105$$

**For Brushless DC Motors:**

$$\tau_m \text{ (msec)} = \frac{R \text{ (Ohms)} \times J \text{ (oz-in-sec}^2\text{)}}{K_t \text{ (oz-in/Amps)} \times K_e \text{ (V/Krpm)}} \times 90.6$$

J = Armature/Rotor Inertia

R = Circuit Resistance

**Thermal Resistance ( $R_{th}$ )** — is useful for predicting the ultimate temperature rise under different loading conditions in order to determine a maximum continuous torque rating. It is usually expressed in the units °C/Watt.

$$R_{th} \text{ (}^\circ\text{C/Watt)} = \frac{\text{Temperature Rise At Rated Load (}^\circ\text{C)}}{\text{Power Losses At Rated Load (Dissipated Watts)}}$$

## Using Performance Data to Calculate $K_t$ and $K_e$

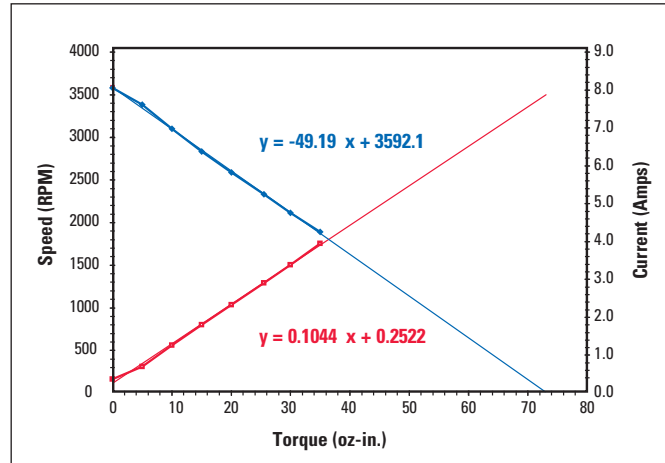
This speed/torque graph demonstrates how the linear equation of the current is used to calculate the Torque Constant ( $K_t$ ) by using the slope "m." The Voltage Constant ( $K_e$ ) can then be calculated.

### In this example:

$K_t = 1/m$  (one divided by the slope (m) of the linear equation for the current)

$K_t = 1/0.1044 \text{ oz-in./Amps} = 9.579 \text{ oz-in./Amps}$

$K_e = K_t/1.3524 \text{ V/Krpm} = 7.0826 \text{ V/Krpm}$   
(or alternatively, multiply  $K_t$  by 0.74)



Performance Graph for 22B2BEBL Brushless DC Motor, Model No. 3502 (24V)

## How to Calculate Torque Constant ( $K_t$ ) and Voltage Constant ( $K_e$ )

Excerpted From Section 8.2 of the [Bodine Handbook](#), Fifth Edition

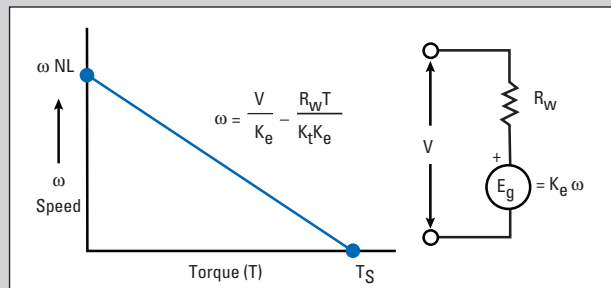
### Voltage Constant ( $K_e$ )

Bodine Electric Company's Handbook shows the speed/torque curve of permanent magnet DC and brushless DC motors. Below is the equation for how to calculate the voltage constant (also known as the function of turns and magnetic flux).

$$V = R_W I + E_g$$

$$\text{or } V = R_W I + K_e \omega$$

where  $K_e$  is a function of turns and magnetic flux.  $K_e$  is called the voltage constant. It is a proportionality constant that relates the generated voltage to the shaft speed ( $\omega$ ).



Speed/torque curve of either a PMDC or BLDC motor

### Torque Constant ( $K_t$ )

Use the graph and equations to determine the torque constant of a permanent magnet DC or brushless DC motor. The torque constant allows you to calculate exactly how much current a motor draws directly from the power supply. The output torque of the motor is directly proportional to the current going into the motor. Since the majority of Bodine products are gearmotors, the user must consider the loading of the gearbox, the duty cycle, and the gear ratio. A gearmotor might not draw nameplate rated current if lightly loaded, or if the gear ratio is very high, e.g.: above 80:1.

If the motor current ( $I$ ) is constant, a proportional torque ( $T$ ) is produced:

$$T = K_t I$$

where  $K_t$  is a function of turns and magnetic flux.  $K_t$  is called the torque constant and is a proportionality constant that relates current to developed torque.

Solving the torque equation for current and substituting the resulting expression for "I" in the voltage equation yields:

$$V = \frac{T R_W}{K_t} + K_e \omega$$

To learn more, please see page 8-8 in the [Bodine Handbook](#).

$R_W$  = Winding Resistance

$I$  = Motor Current

$E_g$  = Back EMF Voltage

$V$  = Voltage applied to motor

$T$  = Output Torque